j, dimensionless diffusion flux onto wall; K_S, rate constant of surface chemical reaction; K_V, rate constant of volume chemical reaction; t, time; X, distance from wall.

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ELECTROTHERMAL ANALOGY IN HEREDITARY MEDIA AND ITS APPLICATION

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UDC 536.2

An electrothermal analogy (ETA) is established for the most common media with a thermal memory. The problem of intensifying thermal perturbations in a system consisting of a plate and a semiinfinite body is examined.

It is currently possible to distinguish a broad range of nonequilibrium physical phenomena in which heat transfer processes cannot be adequately described on the basis of the linear Fourier gradient relation. These cases include the following: heat transfer in liquid helium [1, 2]; heat transfer in media with energy carriers having a low concentration (in low-density gases [3]); heat transfer at low temperatures in crystals and semiconductors by second sound, ballistic phonons, etc. [4-7]; transport phenomena described within the framework of a two-temperature model (in a nonequilibrium gas [8], hot electrons in semiconductors [5, 6]). Mastery of thin-film and laser technologies also requires that allowance be made for memory effects in heat transfer. Similar problems are even more important in mass-transfer processes, where the relaxation time of the processes is several orders greater than in the case of heat transfer. For example, the study [9] described lag effects in various forms of mass transfer (adsorption, drying, heterogeneous catalysis, diffusion processing of porous bodies). Besides transport processes under extreme conditions, it is also possible to see deviations of heat transfer from the Fourier relation under normal conditions for media having a complex structure (polycrystalline materials, polymers, liquid crystals, etc.). Thus, a relaxational effect has been observed [10] in the high-temperature heat capacity of tungsten, this effect being due to the existence of high concentrations of point defects in the metal. The above-mentioned classes of phenomena and materials can be described at the phenomenological level in terms of heat transfer in hereditary media on the basis of integral governing relations (GR) [11, 12] with relaxation functions (RF) $\lambda_1(t)$, $c_1(t)$ for heat flux and internal energy. These functions account for the history of the thermal process. In the particular case of RFs of the form $\lambda_1 = 1 - \exp(-t/\tau_0)$; $c_1(t) = H(t)$, the GRs [11, 12] describe the hypothesis of relaxation of the thermal stress $q + \tau_0 \dot{q} = -\lambda_0 \nabla u$, leading to a hyperbolic heat-conduction equation. The Maxwell relaxation time τ_0 in solids at normal temperature has the value 10^{-9}-10^{-11} sec [13]. Thus, its effect on heat transfer should be considered when a material is subjected to a laser pulse of nanosecond duration [14, 15]. At low temperatures, τ_0 may increase by several orders [5] and have a more significant ef-

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 4, pp. 643-650, October, 1988. Original article submitted May 20, 1987.

0022-0841/88/5504-1166$12.50

fect on heat transfer in the material. The author of [16] used GRs to describe the transfer of heat (and momentum) in polycrystalline metals in an approximation employing several relaxation times. There is also a class of heat and mass transfer processes [in disperse media (DM)] in which lag effects (a memory) [17-21] is manifest due to a difference in the thermophysical characteristics of the materials comprising the DM. In the description of nonsteady transport processes in DMs, these effects appear as asymptotic corrections in the region of large values of time.

Apart from the possibility of describing the phenomena discussed above, interest in heat-conduction processes in media with a thermal memory (hereditary media) stems from the existence of new physical effects in these substances (the existence of two classes of media with finite and infinite rates of heat propagation, the possible existence of weakly absorbing and intensifying thermal media, etc.) which are not observed in normal linear thermal media [12, 22, 23]. Particularly interesting is the class of Maxwell media (with a finite rate of heat propagation), where wave properties are manifest to the greatest extent in heat propagation. Thus, it is necessary to introduce new concepts and ways of thinking into thermal physics. With this in mind, here we introduce an electrothermal analogy for nonsteady processes in the most common thermal media (media with a thermal memory). The effectiveness of the analogy is illustrated in the solution of a specific problem.

As is known (see [24, 25]), the equations describing the propagation of electric current $i(x, t)$ and voltage $e(x, t)$ in a two-conductor line has the following form in the region of the originals and the transforms (with $i(x, t) = e(x, t) = 0$ for $t \leq 0$):

$$\begin{aligned} &\left(L\frac{\partial}{\partial t}+R\right)i=-\frac{\partial e}{\partial x}; \qquad (Lp+R)\,I=-\frac{\partial E}{\partial x};\\ &\qquad\qquad\qquad\qquad\qquad \Rightarrow \\ &\left(C\frac{\partial}{\partial t}+G\right)e=-\frac{\partial i}{\partial x}; \qquad (Cp+G)\,E=-\frac{\partial I}{\partial x}. \end{aligned} \tag{1}$$

Here, L, C, R, and G are the inductance, capacitance, resistance, and leakage per unit length. Equations (1) lead to the following general form for the transforms of voltage E and current I [24]:

$$\begin{aligned} &E(x,\ p)=E(0,\ p)\,\mathrm{ch}\,h(p)\,x-I(0,\ p)\,Z(p)\,\mathrm{sh}\,h(p)\,x;\\ &I(x,\ p)=I(0,\ p)\,\mathrm{ch}\,h(p)\,x-E(0,\ p)\,Z^{-1}(p)\,\mathrm{sh}\,h(p)\,x;\\ &Z(p)=\sqrt{\frac{Lp+R}{Cp+G}};\quad h(p)=\sqrt{(Lp+R)(Cp+G)}. \end{aligned} \tag{2}$$

Here, Z(p) and h(p) are the characteristic impedance of the line and the wave propagation factor. The concepts introduced in Eqs. (2) make it possible to effectively analyze different physical phenomena in lines.

For media with a memory, the GRs [11, 12] and the equation of conservation of internal energy for the unidimensional case (when the energy source is proportional to the temperature, while $u(x, t) = q(x, t) = 0$ at $t \leq 0$) can be written in a form analogous to Eqs. (1):

$$\begin{aligned} &q=-\lambda_0 K_\lambda\frac{\partial u}{\partial x}; \qquad \frac{Q}{\lambda_0 p\Lambda_1(p)}=-\frac{\partial U}{\partial x};\\ &\qquad\qquad\qquad\qquad \Rightarrow \\ &\left(\frac{\lambda_0}{a_0}K_e\frac{\partial}{\partial t}-r_1\right)u=-\frac{\partial q}{\partial x}; \qquad \frac{\lambda_0}{a_0}\left[p^2C_1(p)-r_0\right]U=-\frac{\partial Q}{\partial x}, \end{aligned} \tag{3}$$

$$K_\lambda\frac{\partial u}{\partial x}=\lambda_1(0)\frac{\partial u(x,\ t)}{\partial x}+\int_0^\infty\dot{\lambda}_1(\tau)\frac{\partial u(x,\ t-\tau)}{\partial x}\,d\tau;$$

$$K_e\frac{\partial u}{\partial t}=c_1(0)\frac{\partial u(x,\ t)}{\partial t}+\int_0^\infty\dot{c}_1(\tau)\frac{\partial u(x,\ t-\tau)}{\partial t}\,d\tau;\quad r_0=\frac{a_0}{\lambda_0}r_1 \tag{3a}$$

are integral operators of heat flux and internal energy with the corresponding RFs. It is evident from comparing Eqs. (1) and (3) that the replacement of the transforms E and I by U and Q and substitution of the operators

$$Lp + R \Rightarrow [\lambda_0 p \Lambda_1(p)]^{-1}; \quad Cp \Rightarrow \frac{\lambda_0}{a_0} p^2 C_1(p); \quad G \Rightarrow -r_1 \tag{4}$$

sets up a formal correspondence between the equations of heat conduction in media with a memory and the equations of a two-conductor line, i.e., an electrothermal analogy is established. As in the case of the two-conductor line, double application of the Laplace transform to (3) with respect to t and x (with allowance for zero initial conditions) and subsequent reversion to the region of the originals with respect to x yields the transformation of the general solution of planar one-dimensional problems of heat conduction [26]. The transformation can be written by analogy with (2), where E(x, p), E(0, p), I(x, p), I(0, p) are replaced by U(x, p), U(0, p), Q(x, p), Q(0, p), while the thermal independence Z(p) and the thermal wave propagation factor are:

$$\begin{gathered} Z(p) = \frac{1}{\gamma_0} \frac{S(p)}{K_0(p)} = \frac{1}{\gamma_0}\left(\sqrt{p^3\left[C_1(p) - \frac{r_0}{p^2}\right]\Lambda_1(p)}\right)^{-1}; \\ K_0(p) = \sqrt{\frac{p^2 C_1(p) - r_0}{p\Lambda_1(p)}}; \\ h(p) = K_0(p)/\sqrt{a_0}; \quad S(p) = [p\Lambda_1(p)]^{-1}; \quad \gamma_0 = \sqrt{\lambda_0 \rho_0 c_0}. \end{gathered} \tag{5}$$

Characteristics were introduced for all media with a decaying memory (including normal Fourier media) and for the case of hyperbolic heat conduction equation (standard Maxwell medium). It is evident from (5) that the thermal impedance of the medium and propagation factor depend on the medium's equilibrium thermophysical characteristics and the properties of its memory. In media with a memory, the function $K(p) = K_0^2(p)$ (the analog of $h^2(p)$ in the two-conductor line) is the sum of a second-order polynomial (for Maxwell media) or a first-order polynomial (for Fourier media) in p and one or several partial fractions [23, 26]. Thus, at $|p| \to \infty$ in the right half-plane, h(p) and Z(p) satisfy the condition of transformability [12]. In returning to the originals, this allows us to employ a nonlinear substitution of variables and the associated properties [27].

In the special case of a normal (standard) Fourier medium $\lambda_1(t) = c_1(t) = H(t)$, with the source being proportional to the temperature, the ETA takes the familiar form: $L = 0$; $R = \lambda_0^{-1}$; $C = \rho_0 c_0$; $G = -r_1$; $h(p) = (1/\sqrt{a_0})\sqrt{p - r_0}$; $Z(p) = [\gamma_0\sqrt{p - r_0}]^{-1}$.

In the case of a standard Maxwell medium (hyperbolic heat conduction equation), the ETA takes the form: $L = \tau_0/\lambda_0$; $G = -r_1$; $R = \lambda_0^{-1}$; $C = \rho_0 c_0$; $Z(p) = 1/\gamma_0\sqrt{\tau_0 + (1 + r_0\tau_0)/(p - r_0)}$; $h(p) = (1/\sqrt{a_0})\sqrt{\tau_0 p^2 + p(1 - r_0\tau_0) - r_0}$. It was first established in [28]. A hyperbolic equation for temperature u can be written by introducing effective thermophysical characteristics:

$$\begin{gathered} a_{ef} = a_0/\eta_0; \quad \tau_{ef} = \tau_0/\eta_0; \quad r_{ef} = r_0/\eta_0; \quad \eta_0 = 1 - r_0\tau_0; \\ \frac{\tau_{ef}}{a_{ef}} \frac{\partial^2 u}{\partial t^2} + \frac{1}{a_{ef}} \frac{\partial u}{\partial t} - \frac{r_{ef}}{a_{ef}} u - \frac{\partial^2 u}{\partial x^2} = 0. \end{gathered} \tag{6}$$

It is evident from (6) that $|a_{ef}| > a_0$; here, the case of negative diffusivity is possible (with $r_0 > 1/\tau_0$). In essence, this means that the coefficient with the derivative $\partial u/\partial t$ changes sign and that this term proves to have a reinforcing effect on the propagating temperature field. By analogy with an electrical circuit, we can also introduce a concentrated thermal resistance. The latter is manifest in nonideal contact between two media and is equal to the following (if there are no heat sources at the boundary): $R_t = \Delta U_b/q = 1/\alpha$.

Adoption of the ETA makes it possible to achieve several goals: 1) experimentally model different media with a thermal memory by means of a two-conductor line (with input and output resistances at the ends of the line), as well as experimentally model physical heat propagation processes taking place in the media; 2) apply the approach, method, and results from analysis of the propagation of electromagnetic waves in a two-conductor line to heat propagation processes. Thus, the ETA makes it possible to apply common wave properties (reflection, linear superposition, interference, etc.) to heat-propagation processes in the most common media with a thermal memory. Meanwhile, the same concepts are also applicable to Fourier media (if it is assumed that they are a limiting case of Maxwell media with $w \to \infty$, where w is the rate of propagation of the front of the thermal wave). The ETA allows us to use formulations of certain problems in thermophysics that are analogous to wave formulations. For example, it is possible to formulate the problem of matching the thermal impedances of different media so as to make fullest use of the energy of a travelling thermal wave. Another example would be the problem of matching two thermal media with different impedances by means of a transitional matching layer. It was shown in [22, 23] that given the appropriate mechanism for pumping energy into the medium (by the action of external fields), a thermal medium with a memory may be weakly absorbing or even intensifying. Formally, such a medium is described by inclusion of the volumetric distribution of the energy source, proportional to temperature. The use of such media may lead to the development of different wave-based thermophysical devices that have optical and radiophysical analogs. Below, we examine a thermal system which is an analog of an optical amplifier (laser).

Thermal Amplifier in Media with a Memory. The amplifier is modeled by a system of two bodies (an infinite plate $x \in [0, \ell]$ and a semiinfinite body $x \in [\ell, \infty)$). Ideal thermal contact exists between the bodies. The mathematical model of a problem for such a system is formulated as follows. We assign GRs [11, 12] [with their RFs $\lambda_i(t)$, $c_i(t)$] and internal-energy balance equations (3)-(3a) for each body. Here, we assume that the first medium contains a volumetric source proportional to the temperature $\sigma_1 = r_1 u$, while in the second medium $\sigma_2 = 0$. The nonambiguity conditions include the zero initial conditions $u_i(x, 0) = 0$ (in Maxwell media, we additionally assume that $\partial u_i(x, 0)/\partial t = 0$). The usual conditions are assigned at the boundaries ($x = 0;\ \ell;\ \infty$):

$$u_1(0,\ t) = u_0(t);\ u_1(l,\ t) = u_2(l,\ t);\ q_1(l,\ t) = q_2(l,\ t);$$
$$u_2(\infty,\ t) = q_2(\infty,\ t) = 0. \tag{7}$$

To solve the above problem in the first medium (plate), we use the general form (2), (5) of the transformation of planar one-dimensional problems:

$$U_1(x,\ p) = U_1(0,\ p)\operatorname{ch} h_1(p)x - Q_1(0,\ p)Z_1(p)\operatorname{sh} h_1(p)x;$$
$$Q_1(x,\ p) = Q_1(0,\ p)\operatorname{ch} h_1(p)x - Z_1^{-1}(p)U_1(0,\ p)\operatorname{sh} h_1(p)x;$$
$$h_1(p) = K_{01}(p)/\sqrt{a_1}; \tag{8}$$
$$K_{01}(p) = \sqrt{\frac{p^2C_1(p) - r_0}{p\Lambda_1(p)}};\ Z_1(p) = \frac{1}{\gamma_1}\frac{S_1(p)}{K_{01}(p)};\ S_1(p) = \frac{1}{p\Lambda_1(p)}.$$

In the second medium, we use the general form of the transformation of problems for infinite bodies [22] with allowance for the condition at infinity:

$$U_2(x,\ p) = U_2(l,\ p)\exp[-h_2(p)(x-l)];\ h_2(p) = K_{02}(p)/\sqrt{a_2};$$
$$Q_2(x,\ p) = Q_2(l,\ p)\exp[-h_2(p)(x-l)];\ Z_2(p) = S_2(p)/\gamma_2 K_{02}(p); \tag{9}$$
$$U_2(l,\ p) = Q_2(l,\ p)Z_2(p);\ K_{02}(p) = \sqrt{\frac{pC_2(p)}{\Lambda_2(p)}};\ S_2(p) = \frac{1}{p\Lambda_2(p)}.$$

Nonambiguity condition (7) leads to the following expressions for the boundary functions:

$$U_1(0,\ p) = U_0(p);\ U_1(l,\ p) = \frac{U_0(p)}{D_0(p)};\ Q_2(l,\ p) = \frac{U_1(l,\ p)}{Z_2(p)};$$
$$Q_1(0,\ p) = \frac{U_0(p)}{D_0(p)Z_1(p)}\left[\operatorname{sh} h_1(p)l + \frac{Z_1(p)}{Z_2(p)}\operatorname{ch} h_1(p)l\right]; \tag{10}$$
$$D_0(p) = \operatorname{ch} h_1(p)l + \frac{Z_1(p)}{Z_2(p)}\operatorname{sh} h_1(p)l.$$

In the problem formulated here, we are interested in the case of maximum heat transfer to the second medium. We can solve this problem by the method employed in [26]: expand the hyperbolic functions in (8) and (10) into series in the exponents; introduce partial thermal waves (transmitted and reflected) and reflection coefficients at the boundaries $x = 0$, ℓ; make a detailed analysis of different cases of the solution that is obtained. The solution of the problem can be simplified considerably by using the ETA. It is known from circuit theory (and electrodynamics) that in the case of the equality of the impedances of two lines (media)

$$Z_1(p) = Z_2(p) \tag{11}$$

a wave of electric current (electromagnetic wave) is propagated without reflection at the boundary $x = \ell$ of the lines (media). This corresponds to the maximum transfer of energy by the wave to the second line (medium). The use of condition (11) for the thermal impedances of the media in Eqs. (8-10) leads to the solution

$$\begin{gathered} U_1(x,\ p) = U_0(p)\exp(-h_1(p)x);\ Q_1(x,\ p) = U_1(x,\ p)/Z_1(p); \\ U_1(l,\ p) = U_2(l,\ p) = U_0(p)\exp(-h_1(p)\,l); \\ U_2(x,\ p) = U_0(p)\exp[-h_1(p)\,l - h_2(p)(x-l)];\ Q_2(x,\ p) = U_2(x,\ p)/Z_2(p), \end{gathered} \tag{12}$$

which describes the propagation of a thermal wave without reflection. Condition (11) depends both on the equilibrium thermophysical characteristics of the medium and on the properties of its memory. Exact condition (11) for matching media should be satisfied for all p in the right half-plane of transforms. In practice, approximate matching is obtained using the available numerical parameters of the problem. Here, by making the substitution $p = i\omega$, we change over to spectral representation of the signal and we use the free parameters of thermal media to approximately satisfy (11) in the given frequency range – which is determined by the spectral composition of the input signal. The results apply to all media with a thermal memory. The special case of a normal Fourier medium at $r_0 = 0$ was examined in [13].

Let us examine the most important case – that of a harmonic heating $u_0(t) = \exp(i\omega t)$ in media with matched impedances. Considering that $U_0(p) = (p - i\omega)^{-1}$ and that there is one pole $p = i\omega$ in returning from $U_2(x, p)$ in (12) to the originals, we obtain

$$\begin{gathered} u_2(x,\ t) = \Phi(i\omega)\exp[-\xi_2(\omega)(x-l)]\exp[i\omega(t-(x-l)/w_2(\omega))]; \\ \Phi(i\omega) = A_1(\omega)\exp[i\Phi_1(\omega)];\ A_1(\omega) = \exp[-\xi_1(\omega)\,l];\ \Phi_1(\omega) = -\omega l/w_1(\omega); \\ w_m(\omega) = \omega/\mathrm{Im}\,h_m(i\omega);\ \xi_m(\omega) = \mathrm{Re}\,h_m(i\omega),\ m = 1,\ 2. \end{gathered}$$

Here, $\xi_m(\omega)$, $h_m(\omega)$ are the attenuation factor and the rate of propagation of the thermal wave (TW) in each medium (see [23]). The functions $A_1(\omega)$ and $\Phi_1(\omega)$ are the amplitude–frequency and phase–frequency characteristics (AFC and PFC) of the system with respect to temperature. The AFC shows the ratio of the amplitude of temperature at the boundary $x = \ell$ to the amplitude of the inlet temperature; the PFC shows the phase shift between these values. The expressions ξ_1 and ω_1 fully agree with the analogous expressions in [23]. Thus, the conditions for the functioning of the system as a thermal amplifier supplying a load having a matched impedance coincide with the conditions examined in [23] for the intensification of a thermal wave in a semiinfinite body. The problem of amplification of a broadband thermal signal without phase distortions requires the use of the PFC of the thermal system. The thermal system studied here can be used to measure variable temperature with its preliminary amplification.

It is intuitively clear that the type of heat transfer that occurs in DMs is determined by the model of heat transfer used for the continuous phase. Thus, a DM can be modeled by a certain Fourier medium with an infinite rate of heat propagation, which means that the solutions of heat-transfer problems for DMs should have certain features in common with the solutions for thermal media of the Fourier type [12, 22, 23, 26, 27]. In particular, the phenomenon of the intensification of a TW at low frequencies ($0 < \omega < \omega_{cr}$) may occur in a DM with the release of heat both in volumes of the materials and at the boundaries of the phases comprising the heterogeneous medium. Heat may be released as a result of phase transformations, chemical reactions, and the use of physical means of pumping energy via external fields (electrostatic, etc.). Other phenomena related to the effect of memory and heat release in DMs were examined in [21, 29].

NOTATION

$H(t)$, Heaviside unit function, equal to zero at $t < 0$ and unity at $t \geq 0$; u, q, U, and Q, temperature, heat flux, and their transforms, designated, respectively, by capital letters; λ_0, a_0, ρ_0, c_0, equilibrium values of thermal conductivity, diffusivity, density, and the mass heat capacity of the substance; r_1, proportionality factor in the internal energy source; a_m, $\lambda_m{}^0$, $\rho_m{}^0$, $c_m{}^0$, equilibrium values of diffusivity, thermal conductivity, density, and heat capacity for the two media ($m = 1, 2$); $\gamma = \sqrt{\lambda_m{}^0 \rho_m{}^0 c_m{}^0}$, coefficient of thermal activity of the media; $\lambda_m(t)$, $c_m(t)$, relaxation functions of heat flux and internal energy for the media ($m = 1, 2$); $\Lambda_m(p)$, $C_m(p)$, their Laplace transforms; $Z_m(p)$, $h_m(p)$, impedance and thermal-wave propagation factor for media with a memory; $K_{0m}(p)$, correspondence function for media with a memory ($m = 1, 2$); τ_0, Maxwell relaxation time of the heat flux; α, heat-transfer coefficient; $e_m(x, t)$, volume density of internal energy in the media; e_{m0}, initial values of e_m; σ_m, volume density of internal energy sources; $A_1(\omega)$, $\Phi_1(\omega)$, amplitude–frequency and phase–frequency characteristics of the system.

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